Does the Electoral College Foster Polarization? Turnout and Opposition Demobilization

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March 2026

Abstract

Does the Electoral College foster candidate polarization? I develop a spatial competition model with policy-motivated candidates and endogenous turnout to show that the Electoral College polarizes candidate positions by muting the centripetal force of opposition demobilization. Under a national popular vote, platform moderation reduces turnout among opposition voters in safe states, increasing the electoral return to centrist positioning. Under the Electoral College, safe-state margins are electorally irrelevant, weakening this centripetal force. While standard models without turnout yield no clear ranking of electoral systems, I show that with endogenous turnout the popular vote always produces strictly less equilibrium divergence. The moderation gap increases with the number of uncompetitive states and, under independent shocks, scales with $\sqrt{n}$.

Keywords: Electoral College, popular vote, platform divergence, voter turnout, spatial competition

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1 Introduction

In the five presidential elections from 2000 to 2016, two produced winners who lost the national popular vote. These outcomes—George W. Bush's victory in 2000 and Donald Trump's in 2016—reignited a long-simmering debate about the merits of the Electoral College. Proponents emphasize its protection of federalism and its incentive for candidates to build geographically broad coalitions. Critics counter that it distorts democratic representation by making most states electorally irrelevant: in recent elections, roughly 40 of 51 states and the District of Columbia are classified as "safe" for one party, receiving minimal campaign attention because their outcomes are foregone conclusions. The National Popular Vote Interstate Compact, which would effectively replace the Electoral College with a national popular vote once states representing a majority of electoral votes sign on, has made the prospect of institutional reform concrete.

Yet the debate over the Electoral College centers largely on fairness, voter equality, and campaign resource allocation. A different but consequential question has received less attention: how does the electoral system shape candidates' platform choices? If the aggregation rule—how individual votes translate into electoral victory—affects the ideological positions that candidates adopt, then the choice between the Electoral College and a popular vote has implications not just for who wins, but for what policies voters are offered. This question is especially pressing because the number of competitive states has been shrinking: the geographic sorting of the American electorate, documented by Bishop (2008) and Brown and Enos (2021), means that the Electoral College increasingly concentrates electoral competition in a handful of swing states while rendering the vast majority of the country electorally irrelevant. Understanding how this structural feature of the Electoral College interacts with candidates' positioning incentives requires a formal model that can accommodate multi-state competition with endogenous voter behavior.

This paper develops a formal model to answer this question. I construct a two-candidate spatial competition framework in which candidates are policy-motivated in the Calvert (1985)–Wittman (1977, 1983) tradition, voters engage in expressive voting with heterogeneous costs, and elections take place across multiple states with varying partisan compositions. I compare equilibrium platform divergence—the ideological distance between the two candidates' positions—under the Electoral College and a national popular vote.

The model identifies a mechanism I call *opposition demobilization*. Consider a candidate who moderates her platform, moving it toward the political center. Under the popular vote, this moderation reduces the perceived ideological gap between the candidates for all voters, including opposition voters in the rival party's safe states. These opposition voters, perceiving smaller stakes, become less motivated to turn out. Since every vote counts equally under a popular vote, this opposition demobilization directly improves the moderating candidate's electoral prospects. Under the Electoral College, by contrast, safe-state margins are electorally irrelevant—a blowout and a narrow win both yield the same number of electoral votes. The opposition-demobilization benefit of moderation is therefore invisible to the Electoral College, eliminating a centripetal incentive that the popular vote preserves.

Why does opposition demobilization dominate the countervailing force of base demobilization—the risk that moderation also depresses turnout among one's own supporters? Under quadratic utility, marginal disutility increases with ideological distance, so opposition voters—who are far from the moderating candidate—respond more strongly to moderation than base voters, who are close. The net effect of moderation on the active electorate is therefore centripetal, and this asymmetry is what makes the popular vote systematically more moderating than the Electoral College.

I establish three main results. First, as a benchmark, I show that without endogenous turnout, the comparison between the Electoral College and the popular vote is ambiguous: the popular vote produces more moderation only when safe states are not too polarized (Proposition 1). This benchmark clarifies why previous work in the Calvert–Wittman tradition, which typically abstracts from turnout, finds no clear ranking of electoral systems. Second, with endogenous turnout, the ambiguity is fully resolved: the marginal electoral return to moderation is always strictly larger under the popular vote, leading to strictly less equilibrium platform divergence for any degree of state polarization (Proposition 2). Third, the moderation gap grows with the number of electorally uncompetitive states: in the limit where each state is safe for one party, the ratio of marginal returns reaches $\sqrt{n}$ with independent state shocks (Corollary 1).

The theoretical upper bound of $\sqrt{n}$, however, assumes independent state-level shocks. Calibrating to U.S. presidential elections (1972–2024), I estimate the average cross-state swing correlation at $\rho \approx 0.72$, which reduces the marginal-return ratio from $\sqrt{51} \approx 7.1$ to roughly 1.18. The qualitative result—that the popular vote always moderates more—is unconditional, but the quantitative magnitude is more modest than the upper bound suggests. The advantage is meaningful and growing: cross-state correlation has declined from 0.77 in 1972–1992 to 0.60 in 2016–2024, reflecting increasing geographic sorting.

The paper makes three contributions. First, I extend the Calvert (1985)–Wittman (1977, 1983) framework of policy-motivated candidate divergence (see also Besley and Coate 1997; Krasa and Polborn 2012) to a multi-state setting where the aggregation rule—Electoral College versus popular vote—is an institutional variable. Single-constituency models predict divergence but cannot distinguish between electoral systems, since the aggregation rule is trivially defined with a single constituency. Second, building on the literature on turnout and platform choice (Glaeser, Ponzetto, and Shapiro 2005; Herrera, Levine, and Martinelli 2008; Feddersen and Sandroni 2006), I identify a countervailing channel—opposition demobilization—and show that its electoral relevance depends on the aggregation rule. In a single constituency, base mobilization and opposition demobilization are symmetric forces; the multi-state setting with different aggregation rules makes them asymmetric, because the Electoral College suppresses safe-state margins where the opposition-demobilization effect operates. Third, relative to the literature on electoral institutions and political competition (Strömberg 2008; Callander 2005; Lizzeri and Persico 2001; Matakos, Troumpounis, and Xefteris 2016; Myerson 1993), I focus on platform positioning and provide a micro-foundation for why aggregation rules affect polarization.

The paper proceeds as follows. Section 2 presents the model. Section 3 establishes the benchmark result without endogenous turnout. Section 4 presents the main result with turnout and its generalization to n states. Section 5 discusses extensions. Section 6 discusses empirical implications and limitations. Section 7 concludes.

2 The Model

2.1 Environment

Two candidates, L and R, compete over a unidimensional policy space $X = \mathbb{R}$. Each candidate simultaneously announces and commits to a policy platform: L chooses $x_L \in \mathbb{R}$ and R chooses $x_R \in \mathbb{R}$. Candidates are policy-motivated. Candidate L has an ideal point at $-\beta$ and candidate R has an ideal point at β, where $\beta > 0$ represents the degree of ideological divergence between the candidates' preferences. If policy x is implemented, candidate j's utility is

$$U_L(x) = -(x+\beta)^2, \qquad U_R(x) = -(x-\beta)^2.$$

The winning candidate implements her announced platform.

There are three states, $s \in \{\ell, m, r\}$, each containing a continuum of voters with unit mass. Voters in state s have ideal points distributed according to $N(\mu_s, \sigma^2)$, where

$$\mu_\ell = -\mu, \qquad \mu_m = 0, \qquad \mu_r = \mu,$$

with $\mu > 0$ and $\sigma > 0$. State ℓ is left-leaning, state m is the swing state, and state r is right-leaning.

2.2 Voter Behavior

A voter at ideal point i derives utility $u_i(x) = -(x-i)^2$ from implemented policy x. The voter prefers the candidate whose platform is closer to her ideal point: she prefers L if $i < \bar{x} \equiv (x_L + x_R)/2$ and prefers R if $i > \bar{x}$.

Voters engage in expressive voting with heterogeneous costs. Each voter independently draws a private voting cost $c \sim U[0, \bar{c}]$. A voter turns out and votes for her preferred candidate if and only if the perceived difference between the two candidates exceeds her cost:

$$|u_i(x_L) - u_i(x_R)| \geq c. \tag{1}$$

The utility difference simplifies to $2\Delta|\bar{x} - i|$, where $\Delta = x_R - x_L > 0$ is the platform divergence. The turnout probability is therefore

$$\tau(i) = \min\left(\frac{2\Delta|\bar{x} - i|}{\bar{c}},\ 1\right). \tag{2}$$

Assumption 1. *Cost dispersion is sufficiently large that $\tau(i) < 1$ for all voters, so that turnout takes the linear form $\tau(i) = \alpha \cdot 2\Delta|\bar{x} - i|$ where $\alpha = 1/\bar{c} > 0$.*

Under this assumption, voters who perceive little difference between candidates—those near the midpoint $\bar{x}$—are least likely to turn out, while voters far from the midpoint, with strong preferences, are most likely to vote. The linear dependence on $|\bar{x} - i|$ reflects a key property of quadratic utility: marginal disutility *increases* with distance ($|u_i'(x)| = 2|x - i|$), so distant voters are more sensitive to platform changes than nearby voters. This creates a direct link between platform positioning and the composition of the active electorate.

2.3 Election Uncertainty

Each state s receives an independent popularity shock $\omega_s \sim N(0, \sigma_\omega^2)$, realized after platform choices. The shock captures state-specific factors—local campaign quality, weather, economic conditions—that affect the net vote margin but are not foreseeable at the time of platform choice. Let $V_{j,s}$ denote the expected votes for candidate j in state s. Candidate L wins state s if $V_{L,s} - V_{R,s} + \omega_s > 0$.

2.4 Electoral Systems

Under the *Electoral College* (EC), L wins by carrying a majority of states. Under the *national popular vote* (PV), L wins if total votes across all states favor her.

2.5 Timing and Solution Concept

The timing is: (1) candidates simultaneously choose platforms; (2) state shocks realize; (3) voters observe platforms, realize costs, and turn out; (4) the election outcome is determined; (5) the winner implements her platform. The solution concept is Nash equilibrium. Nash equilibrium is the natural choice for a simultaneous-move complete-information game; candidates choose platforms to maximize expected utility given the opponent's strategy, and the unique symmetric equilibrium I characterize is the fixed point of the best-response mapping.

2.6 Comments on the Model

Several modeling choices deserve discussion.

Why expressive voting? I adopt expressive (consumption-value) voting rather than pivotal (instrumental) voting because the pivotal calculus produces negligible turnout in large electorates, whereas observed turnout rates are substantial. The expressive framework, following Riker and Ordeshook (1968), generates meaningful turnout that responds to the ideological distance between candidates—the empirically relevant pattern. Group-based rational-choice models such as Feddersen and Sandroni (2006) and Coate and Conlin (2004) provide an alternative micro-foundation for substantial turnout through ethical or duty-based motivations. The opposition-demobilization channel would likely survive in such frameworks: the key driver is that the utility difference $|u_i(x_L) - u_i(x_R)|$ determines relative mobilization, and this utility-difference component is present in group-based models as well, since groups with smaller perceived stakes have weaker motivation to mobilize. Formally verifying this conjecture in a group-pivotal framework is a valuable direction for future work, and I note it as a limitation of the present analysis.

Why linear turnout? The linear turnout function $\tau(i) = \alpha \cdot 2\Delta|\bar{x} - i|$ is a consequence of quadratic voter preferences combined with uniform cost dispersion. This specification delivers a key analytical property: the vote margin in each state depends on the state mean μ_s but not on the variance σ^2, yielding clean closed-form results that make the comparison between electoral systems transparent. This distributional insensitivity is a useful feature: it means the results do not depend on the within-state dispersion of voter preferences, only on how state means differ across states. I discuss robustness to alternative utility curvature in Section 5.

Why independent state shocks? With a purely common national shock, the Electoral College and popular vote become equivalent under linear turnout—a single shock affects every state identically, so aggregating votes provides no additional information beyond looking at a single state. Independent (or partially independent) state-level shocks are what make the aggregation rule consequential. I analyze correlated shocks as an extension in Section 5, where I also provide empirical calibration using U.S. presidential election data.

3 Benchmark: No Turnout Effects

Before analyzing endogenous turnout, I establish a benchmark where all voters turn out with certainty, corresponding to the standard Calvert–Wittman framework in a multi-state setting. This benchmark serves two purposes: it isolates the role of turnout in the comparison between electoral systems, and it explains why previous work finds the comparison ambiguous.

3.1 Equilibrium Characterization

Under the popular vote, a candidate's platform shift moves the national vote margin through all states simultaneously: every voter affected by the shift contributes to the aggregate. Under the Electoral College, what matters is winning states: a shift helps only insofar as it flips competitive states. Without turnout effects, these channels pull in opposite directions depending on state polarization, because the Electoral College ignores safe-state margins while the popular vote aggregates both informative swing-state margins and noisy safe-state margins.

Let P denote L's winning probability under either system. Candidate L's expected utility is $\mathbb{E}[U_L] = P \cdot [-(x_L + \beta)^2] + (1 - P) \cdot [-(x_R + \beta)^2]$. At a symmetric profile ($x_L = -x^*$, $x_R = x^*$, $P = 1/2$), the first-order condition $\partial \mathbb{E}[U_L]/\partial x_L = 0$ reduces to

$$\Gamma \cdot 4\beta x^* = \beta - x^*, \tag{3}$$

where $\Gamma \equiv \partial P/\partial x_L|_{\text{sym}} > 0$ is the *marginal electoral return to moderation* at the symmetric profile. The left side is the electoral benefit (higher win probability times the stake $4\beta x^*$); the right side is the policy cost.

Without endogenous turnout, the vote share for L in state s is $F_s(\bar{x}) = \Phi((\bar{x} - \mu_s)/\sigma)$. Since $\bar{x} = 0$ at any symmetric profile, the marginal return Γ_j^{NT} is a constant that depends on $(\mu, \sigma, \sigma_\omega)$ but not on x^*. The first-order condition is therefore linear in x^*:

$$x_j^{*\mathrm{NT}} = \frac{\beta}{1 + 4\beta\,\Gamma_j^{\mathrm{NT}}}. \tag{4}$$

Since x^* is strictly decreasing in Γ, the system with the larger marginal return produces less divergence. The symmetric equilibrium is the unique Nash equilibrium; uniqueness is verified in the Online Appendix.

3.2 The Ambiguity Result

Proposition 1 (Benchmark: No Turnout Effects). *Without endogenous turnout, in the symmetric equilibrium:*

(a) For any $\sigma_\omega > 0$, PV produces less divergence than EC when states are sufficiently similar (small μ/σ).

(b) If state shocks are small enough that safe states are genuinely safe ($\sigma_\omega < \bar{\sigma}_\omega$), there exists a threshold $\bar{\rho}(\sigma_\omega) > 0$ such that EC produces less divergence when $\mu/\sigma > \bar{\rho}$.

(c) If state shocks are large ($\sigma_\omega \geq \bar{\sigma}_\omega$), PV produces less divergence for all $\mu/\sigma \geq 0$.

The critical shock standard deviation $\bar{\sigma}_\omega$ solves $\Phi(1/\bar{\sigma}_\omega)^2 + (1 - \Phi(1/\bar{\sigma}_\omega))^2 = 1/\sqrt{3}$, giving $\bar{\sigma}_\omega \approx 1.94$. Proof in Online Appendix.

Proposition 1 clarifies the basic tension. When states are similar (part a), aggregating binary win-or-lose outcomes is less responsive to platforms than counting all votes, favoring PV. When states are highly polarized and shocks are small (part b), safe-state margins become informationally inert: platform changes barely move the vote margin there. Under PV, random shocks from these inert states dilute the connection between platforms and outcomes; the EC, by discarding safe-state margins, concentrates the election on the responsive swing state. The benchmark's ambiguity turns entirely on the assumption of exogenous turnout. With endogenous turnout, platforms also change *who votes*—a channel that, as I show next, makes vote margins far more responsive to candidates' positions under PV. A more detailed interpretation of the benchmark is provided in the Online

Appendix.

4 Main Result: Turnout and Platform Divergence

4.1 The Opposition Demobilization Mechanism

I begin with the intuition that drives the main result before presenting the formal analysis.

With endogenous turnout, the expected vote margin for L in state s takes a simple form.

Lemma 1. *Under Assumption 1, the expected vote margin for L in state s is*

$$M_s \equiv V_{L,s} - V_{R,s} = 2\alpha\Delta(\bar{x} - \mu_s). \tag{5}$$

Proof in Online Appendix.

The sensitivity of the vote margin to L's platform at a symmetric profile is:

$$D_s \equiv \left.\frac{\partial M_s}{\partial x_L}\right|_{\text{sym}} = 2\alpha(\mu_s + x^*). \tag{6}$$

This equation reveals the mechanism. When candidate L moderates (moves x_L rightward):

- *Swing state* ($D_m = 2\alpha x^*$): Moderation helps L by shifting the midpoint rightward, capturing more centrist voters.
- *Own safe state* ($D_\ell = 2\alpha(x^* - \mu)$): When $\mu > x^*$, moderation *hurts* L in her safe state by demobilizing base voters who perceive a smaller ideological gap.
- *Opposition safe state* ($D_r = 2\alpha(x^* + \mu)$): Moderation *helps* L by demobilizing opposition voters in the rival's stronghold.

The opposition effect always dominates the base effect: $D_\ell + D_r = 4\alpha x^* > 0$. This dominance traces to the increasing marginal disutility of quadratic preferences: opposition voters in state r, who are far from L, are more sensitive to L's moderation than base voters in state ℓ, who are close to L. Under the Electoral College, only D_m matters—safe-state margins are electorally irrelevant. Under the popular vote, safe-state margins count, and the net effect of moderation across safe states is unambiguously positive.

4.2 Marginal Electoral Returns

The intuition from the preceding section can be stated as a comparison of signal-to-noise ratios. Under the popular vote, the total margin sensitivity aggregates D_s across all states: moderation moves the national margin through every state simultaneously. The aggregate shock, however, grows only as $\sqrt{n}$ (for independent shocks), so the signal-to-noise ratio is amplified by the aggregation. Under the Electoral College, moderation moves margins only in the swing state, and only the swing state's shock matters. The popular vote therefore offers a stronger signal-to-noise ratio: it extracts more information from the same set of state-level outcomes.

Formally, under PV, the winning probability aggregates all state margins. At a symmetric profile:

$$\Gamma_{\text{PV}} = \frac{\phi(0)}{\sqrt{3}\,\sigma_\omega} \cdot \sum_s D_s = \sqrt{3}\,\gamma_0\,x^*, \tag{7}$$

where $\gamma_0 \equiv 2\alpha/(\sigma_\omega\sqrt{2\pi})$ and I used $D_\ell + D_m + D_r = 6\alpha x^*$.

Under EC, applying the chain rule to the EC winning probability at a symmetric profile yields (derivation in Online Appendix):

$$\Gamma_{\text{EC}} = \gamma_0\, f(\lambda)\, x^*, \qquad f(\lambda) \equiv e^{-\lambda^2/2} + \Phi(\lambda)^2 + (1-\Phi(\lambda))^2, \tag{8}$$

where $\lambda \equiv 4\alpha x^* \mu/\sigma_\omega$. The function f captures how state polarization shapes the EC return: the first term is the residual competitiveness of safe states; the second two terms capture the swing state's enhanced pivotality. As μ increases, f decreases from $f(0) = 3/2$

to $\lim_{\lambda\to\infty} f(\lambda) = 1$; in the safe-state limit, only D_m matters and $\Gamma_{\text{EC}} \to \gamma_0 x^*$.

Under both systems, the marginal return takes the form $\Gamma_j = \gamma_j\, x^*$ for a system-specific constant $\gamma_j > 0$. Substituting into the first-order condition (3) yields a quadratic in x^* with a unique positive root:

$$x_j^* = \frac{-1 + \sqrt{1 + 16\beta^2\gamma_j}}{8\beta\gamma_j}, \tag{9}$$

where $\gamma_{\text{PV}} = \sqrt{3}\,\gamma_0$ and $\gamma_{\text{EC}} = \gamma_0\, f(\lambda)$. Since x_j^* is strictly decreasing in γ_j, the system with the larger marginal return produces less divergence.

4.3 Comparing Divergence

Proposition 2 (Main Result). *Under Assumption 1, for any degree of state polarization $\mu \geq 0$:*

$$\frac{\Gamma_{\text{PV}}}{\Gamma_{\text{EC}}} = \frac{\sqrt{3}}{f(\lambda)} > 1.$$

Consequently, $x^*_{\text{PV}} < x^*_{\text{EC}}$: the popular vote always produces strictly less platform divergence. The ratio is strictly increasing in μ, ranging from $2\sqrt{3}/3 \approx 1.15$ (all states identical) to $\sqrt{3} \approx 1.73$ (safe-state limit). Proof in Online Appendix.

Proposition 2 is the paper's central result. Its content can be understood at three levels.

First, the *qualitative* claim is that the popular vote always produces strictly less platform divergence than the Electoral College, regardless of how polarized the states are. This is an unconditional result: it holds for any values of the model parameters $(\beta, \alpha, \mu, \sigma, \sigma_\omega)$. The ambiguity of the benchmark (Proposition 1) is fully resolved once turnout is endogenous. The opposition-demobilization channel is sufficiently strong to overcome any potential advantage that the Electoral College might derive from concentrating the election on competitive states.

Second, the *monotonicity* of the ratio in μ connects to the empirical trend of geographic sorting. As states become more polarized—more solidly partisan—the Electoral College's marginal return to moderation declines because safe-state margins become increasingly irrelevant. The popular vote's marginal return, however, continues to aggregate margin sensitivities from all states, including the increasingly one-sided safe states where opposition demobilization operates most powerfully. The ratio $\Gamma_{\text{PV}}/\Gamma_{\text{EC}}$ therefore increases from 1.15 to 1.73 as μ grows. In an era of increasing geographic sorting—where the number of genuinely competitive states has declined from roughly 20 in the 1990s to fewer than 10 in recent cycles—this monotonicity takes on practical significance.

Third, the *magnitude* of the ratio deserves careful interpretation. In the three-state model, the ratio ranges from 1.15 to 1.73, meaning the popular vote provides 15% to 73% more marginal electoral return to moderation. The resulting reduction in equilibrium divergence follows from equation (9) and depends on all parameters, but the direction is unambiguous. The ratio is a lower bound on the divergence gap in models with more states, as the following section shows. The symmetric equilibrium is unique under both electoral systems; see the Online Appendix.

4.4 Generalization to n States

The model extends naturally to $n = 2K + 1$ states with one swing state and K safe states on each side. In the safe-state limit, only the swing state is competitive under EC, so Γ_{EC} is independent of n. Under PV, the total margin sensitivity $\sum_s D_s$ grows as n while the aggregate shock grows as $\sqrt{n}$, yielding:

Corollary 1. *In the safe-state limit with n states and independent shocks, $\Gamma_{PV}^{(n)} = \sqrt{n}\,\Gamma_{EC}$, and $x^*_{PV}(n) \to 0$ as $n \to \infty$ while x^*_{EC} is independent of n. Proof in Online Appendix.*

The $\sqrt{n}$ scaling is best understood as a theoretical upper bound: it requires independent state-level shocks and all states except one being perfectly safe. With correlated shocks, the ratio generalizes to $\sqrt{n/(1 + (n - 1)\rho)}$ where ρ is the intra-class correlation (Section 5.3). For empirically estimated $\rho \approx 0.72$, this reduces the ratio from $\sqrt{51} \approx 7.1$ to approximately 1.18. Nevertheless, the comparative static carries substantive content: the

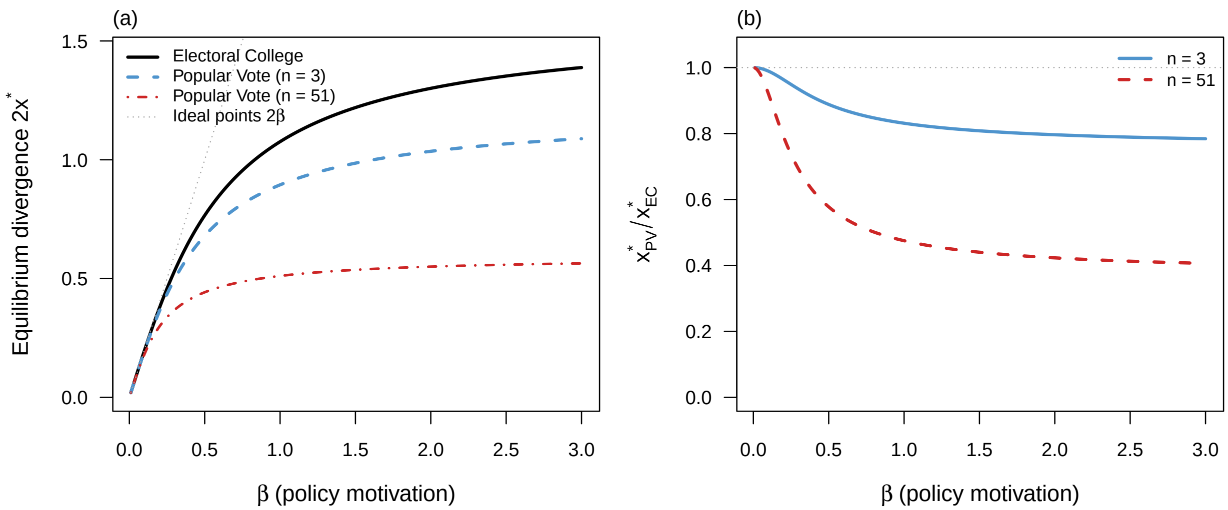

Figure 1: (a) Equilibrium platform divergence under the Electoral College and the national popular vote. The gray dotted line shows the candidates' ideal points 2β. (b) Ratio of popular vote to Electoral College equilibrium divergence. Parameters: $\alpha = 1$, $\sigma_\omega = 2$.

moderating advantage grows with the number of electorally uncompetitive states, and the declining cross-state correlation (ρ fell from 0.77 in 1972–1992 to 0.60 in 2016–2024) implies that this advantage has been growing over time.

4.5 Numerical Illustration

Figure 1 plots the equilibrium divergence $2x^*$ and the ratio $x^*_{\text{PV}}/x^*_{\text{EC}}$ as functions of the policy motivation parameter β. The popular vote always produces less divergence, and the gap widens substantially with n: at $n = 51$, PV divergence is roughly half the EC divergence under the independent-shock assumption. Both systems exhibit the Calvert–Wittman pattern—divergence increases with policy motivation but never reaches the candidates' ideal points—with the popular vote systematically closer to the median.

4.6 Comparative Statics

The closed-form equilibrium (9) yields clean comparative statics with intuitive political interpretations.

Greater policy motivation β increases divergence under both systems: as candidates care more about implementing their preferred policies, they sacrifice more winning probability to stake out ideologically distinct positions. As $\beta \to 0$, both systems converge to zero divergence (Downsian convergence), and as $\beta \to \infty$, both approach their ideal points. This parameter connects to the candidate-selection literature: if partisan primaries select candidates with more extreme preferences (higher β), the model predicts greater divergence under both systems, with the gap between EC and PV preserved.

Higher turnout sensitivity α reduces divergence under both systems, but proportionally more under PV, because the opposition-demobilization channel grows stronger when voters are more responsive to ideological distance. This comparative static connects to voter-engagement reforms: policies that lower voting costs (automatic registration, early voting, mail-in balloting) effectively increase α, and the model predicts that such reforms would reduce divergence more under a popular vote than under the Electoral College.

Greater electoral uncertainty σ_ω increases divergence under both systems: when election outcomes are dominated by random shocks, candidates can afford to stake out extreme positions because platforms matter less for the outcome. In the limit $\sigma_\omega \to \infty$, candidates announce their ideal points under both systems, as the election becomes a coin flip regardless of platforms. This comparative static relates to Calvert's (1985) original insight that convergence requires sufficient electoral uncertainty—candidates must believe that platform choices affect outcomes.

5 Extensions and Robustness

5.1 Asymmetric Electorates

The main analysis assumes symmetric partisan geography: each candidate has one safe state and one shared swing state. In practice, partisan geography is rarely symmetric—one party may have more safe states, or the safe states may be of different sizes and degrees of partisan lean.

In the safe-state limit, the $\sqrt{n}$ result extends to asymmetric state means. When the net partisan lean $S_\mu = \sum_s \mu_s \neq 0$, the ratio of marginal electoral returns becomes $\sqrt{n} + S_\mu/(\sqrt{n}|x_L|)$ for candidate L and $\sqrt{n} - S_\mu/(\sqrt{n}\, x_R)$ for R: asymmetry tilts the opposition-demobilization channel toward the minority party's candidate but does not eliminate it for either. The popular vote produces more moderation for *both* candidates as long as the partisan imbalance is not too extreme. Intuitively, even when one party dominates the map, the minority party's candidate still benefits from demobilizing opposition voters in the majority party's numerous safe states, and the majority party's candidate still benefits from demobilizing opposition voters in the minority's fewer but real safe states. The formal derivation, including the exact condition on partisan imbalance, is in the Online Appendix.

For the contemporary United States, where both parties have substantial numbers of safe states (roughly 20 each), the condition is easily satisfied. The asymmetric-electorate extension therefore reinforces the main result rather than qualifying it.

5.2 Role of Utility Curvature

The opposition demobilization result depends on the curvature of voter preferences, not just on the turnout specification. Under a general loss function $v(|x - i|)$ with $v' > 0$, the perceived difference $|v(|x_R - i|) - v(|x_L - i|)|$ determines turnout. With the quadratic $v(d) = d^2$, marginal disutility $v'(d) = 2d$ increases with distance, so voters far from a candidate are most sensitive to that candidate's platform shift; this is what makes opposition demobilization dominate base demobilization.

For the power family $v(d) = d^p$: when $p > 1$, marginal disutility increases with distance and the mechanism is preserved; when $p \leq 1$, marginal disutility does not increase, potentially attenuating or reversing the gap. The curvature parameter p is therefore central to the result, not a peripheral robustness concern.

The quadratic specification ($p = 2$) is standard in the Calvert–Wittman tradition. The assumption of convex disutility is consistent with loss aversion, but Proposition 2 is proven under quadratic utility and should be understood as conditional on $p > 1$. Whether the threshold at which the mechanism reverses is empirically relevant remains an open question.

5.3 Correlated Shocks

The model assumes independent state-level shocks. Suppose instead that shocks have a common component: $\omega_s = \eta + \varepsilon_s$, where $\eta \sim N(0, \sigma_\eta^2)$ is a national shock and $\varepsilon_s \sim N(0, \sigma_\varepsilon^2)$ is state-specific, with all components mutually independent. In the safe-state limit, the ratio of marginal returns generalizes to

$$\frac{\Gamma_{\text{PV}}}{\Gamma_{\text{EC}}} = \sqrt{\frac{n}{1 + (n-1)\rho}},$$

where $\rho \equiv \sigma_\eta^2/\sigma_\omega^2$ is the intra-class correlation of state shocks. When $\rho = 0$ (purely idiosyncratic), the ratio is $\sqrt{n}$, recovering Corollary 1. When $\rho = 1$ (purely common), the ratio is one and the two systems are equivalent—a single national shock affects every state identically, so aggregating votes provides no additional information. For any $\rho < 1$, the popular vote still produces strictly more moderation, but the gap narrows as the common component grows.

Empirical calibration. I estimate the empirically relevant ρ using state-level two-party vote shares from the 14 U.S. presidential elections between 1972 and 2024 (Dave Leip Atlas). For each consecutive pair of elections, I compute the swing in each state's Democratic two-party vote share, then calculate the pairwise correlation of swings across all 51 states (including DC). The average pairwise swing correlation is $\hat{\rho} = 0.72$. This estimate is consistent with published estimates from election-forecasting models (Linzer 2013) and reflects the substantial common component in presidential election swings: when one party gains nationally, it tends to gain in most states.

The implied moderation ratio at $\hat{\rho} = 0.72$ is $\sqrt{51/37.0} \approx 1.18$. The popular vote still produces a larger marginal return to moderation, but the advantage is substantially smaller than the $\sqrt{51} \approx 7.1$ upper bound. The calibrated ratio means that the popular vote provides roughly 18% more marginal electoral return to moderation than the Electoral College for empirically realistic parameters.

The time trend is informative: the average swing correlation declined from 0.77 (1972–1992) to 0.69 (1996–2012) to 0.60 (2016–2024), consistent with the geographic sorting documented by Brown and Enos (2021). The implied marginal-return ratio has grown from approximately 1.14 to 1.28, and will widen further if the trend toward greater geographic sorting continues.

5.4 Heterogeneous State Sizes

The model assigns equal population to each state. In practice, U.S. states vary enormously in population: California has roughly 68 times the population of Wyoming, and the ratio of largest to smallest state population has been growing over time.

The opposition demobilization mechanism extends to heterogeneous state sizes. Under the popular vote, each state's margin is weighted by its voting population, so large states contribute more to the national margin. The key property—that opposition demobilization dominates base demobilization—depends on the *partisan lean* of each state, not on its population size. A large safe state and a small safe state of the same partisan lean contribute the same *per-voter* margin sensitivity; the large state simply contributes more in aggregate because it has more voters. Under the Electoral College, heterogeneous state sizes alter which states are pivotal (large swing states are more pivotal than small swing states) but do not change the fundamental irrelevance of safe-state margins under winner-take-all.

One subtlety is that heterogeneous state sizes reduce the "effective n" in the model.

With n equal-sized states, the $\sqrt{n}$ ratio reflects the information-aggregation advantage of counting all votes. With unequal sizes, the effective number of independent electoral units is smaller than n, analogous to the inverse Herfindahl index of population shares. For the United States, where the top 10 states contain over half the population, the effective n is substantially below 51. This further attenuates the $\sqrt{n}$ upper bound relative to the equal-population case.

I also note that the two-senator bonus in the Electoral College, which overweights small states relative to their population, interacts with the mechanism. Small states tend to be safe states (both parties have safe small states), so the overweighting of small states amplifies the Electoral College's tendency to discard safe-state margins. A formal treatment of population-weighted margins with the two-senator bonus is a natural extension but is beyond the scope of the present model.

6 Discussion

6.1 Empirical Implications

The model generates several testable predictions, each with distinct identification challenges.

Cross-system comparison. Platform divergence should be greater in elections decided by winner-take-all rules with many safe constituencies than in elections decided by aggregate vote shares. The most direct test would compare presidential systems that use electoral colleges (the United States; historically, Argentina and Finland) with those using popular votes (France, Brazil, most Latin American democracies). The challenge is that electoral systems differ along many dimensions simultaneously—ballot structure, party system, constitutional powers—making clean cross-national comparisons difficult. Within-country variation, such as the National Popular Vote Interstate Compact taking effect, would provide a more compelling natural experiment, though it has not yet occurred.

Time-series within the United States. As the number of competitive states has declined over recent decades, the model predicts that the divergence gap between what candidates would choose under EC versus PV has widened. Since we observe only EC outcomes, testing this requires either a counterfactual model or a measure of how much candidates invest in appealing to safe-state voters. One indirect test: if opposition demobilization matters, we should observe that safe-state turnout is more responsive to platform moderation than swing-state turnout, since safe-state voters have more extreme preferences and are thus more sensitive to ideological distance. State-level turnout data combined with platform-positioning measures (such as DW-NOMINATE scores applied to presidential candidates) could test this differential-response prediction.

Opposition-demobilization channel. The most distinctive prediction is that moderation by one candidate should differentially reduce turnout in the opposing party's safe states relative to the swing state. This prediction distinguishes the opposition-demobilization mechanism from base-mobilization models (GPS), which predict that *extremism* drives turnout in the candidate's own safe states. Testing this requires exogenous variation in platform positioning—a candidate who unexpectedly moderates relative to expectations—and state-level or county-level turnout data disaggregated by partisanship.

6.2 Alternative Mechanisms and Limitations

Several mechanisms not modeled in this paper could interact with the opposition-demobilization channel.

Primary elections. Partisan primaries create centrifugal pressure by selecting candidates who appeal to the party base. This force operates under both the Electoral College and the popular vote. The key question is whether primaries differentially interact with the two systems. If primaries select more extreme candidates precisely because the Electoral College weakens general-election moderation incentives, then the total effect of switching to a popular vote would be larger than my model suggests, since it would affect both the nomination stage and the general election. I treat primary selection as exogenous (β is fixed), which means my estimates of the divergence gap are conservative to the extent that primaries are responsive to general-election incentives.

Campaign spending. Strömberg (2008) shows that the Electoral College concentrates campaign spending in swing states. Spending and platform positioning are likely complements: campaign spending amplifies whatever platform message a candidate adopts. If spending is more effective at mobilizing voters than at persuading them, the interaction reinforces the opposition-demobilization channel by making safe-state voters more aware of platform differences.

Multi-candidate competition, valence dimensions, and dynamic considerations are limitations shared with most spatial-competition models. I note them explicitly as directions for extension rather than weaknesses specific to the present framework.

7 Conclusion

The choice between the Electoral College and a national popular vote has consequences for candidates' ideological positioning that go beyond the familiar concerns about fairness and campaign attention. The key mechanism identified in this paper is *opposition demobilization*: under the popular vote, platform moderation demobilizes opposition voters in the rival party's safe states, creating a centripetal incentive that the Electoral College suppresses. The marginal electoral return to moderation is always strictly larger under the popular vote (Proposition 2), with the gap growing as the number of electorally uncompetitive states increases (Corollary 1).

The quantitative magnitude of this gap depends on the correlation structure of state-level electoral shocks. The theoretical upper bound of $\sqrt{n}$ assumes independent shocks; with the empirically estimated cross-state correlation of $\hat{\rho} \approx 0.72$, the marginal-return ratio for the United States is approximately 1.18. This is a meaningful but modest effect. The time trend, however, points toward a growing gap: as geographic sorting has increased, the correlation of state-level swings has declined, and the moderating advantage of the popular vote has strengthened. If the trend continues, the policy-positioning consequences of the electoral system will become increasingly salient.

The model has clear limitations that merit acknowledgment. The expressive-voting framework, while generating empirically realistic turnout patterns, has not been verified against group-based rational-choice models that produce substantial turnout through ethical motivations. The result depends on convex disutility ($p > 1$ in the CES utility family), a standard but untested assumption. The model treats candidate preferences as exogenous, abstracting from the primary-election process that selects candidates. And the equal-population assumption, while analytically convenient, understates the heterogeneity of the actual U.S. Electoral College.

Despite these limitations, the model delivers a clear, qualitatively robust result: the Electoral College, by rendering safe-state margins electorally irrelevant, eliminates a moderating incentive that the popular vote preserves. In an era when the vast majority of states are electorally safe, this structural feature of the Electoral College deserves attention alongside the more familiar debates about fairness and representation.

Two directions merit further investigation. First, incorporating strategic campaign spending would allow the model to analyze the interaction between resource allocation and platform positioning under different aggregation rules—two dimensions that likely reinforce each other. Second, extending the framework to endogenize candidate selection through primary elections could address whether the centripetal pressure of the popular vote in general elections is offset by centrifugal pressure in nomination contests, a question with direct implications for the net effect of electoral reform on policy outcomes.

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Online Appendix for
"Electoral College vs. Popular Vote: Turnout and the Platform Divergence Gap"

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February 27, 2026

This appendix contains supplementary material for the main text: (A.1) proof of Lemma 1, (A.2) benchmark marginal returns, proof of Proposition 1, and extended interpretation, (A.3) proof of Proposition 2, (A.4) proof of Corollary 1, (A.5) the second-order conditions, (A.6) equilibrium uniqueness, (A.7) the asymmetric states extension, (A.8) derivation of the exact EC marginal return and monotonicity of f, (A.9) additional comparative statics, (A.10) correlated shocks (general case), and (A.11) numerical verification of equilibrium uniqueness. Equation, proposition, and assumption numbers without a prefix (e.g., Proposition 2, equation (3)) refer to the main text.

1 Proof of Lemma 1

I derive the expected vote margin $M_s = V_{L,s} - V_{R,s}$ for candidate L in state s under Assumption 1.

Write $f_s(i) = \phi((i - \mu_s)/\sigma)/\sigma$ for the density of voters in state s. A voter at ideal point i votes for L if $i < \bar{x}$ and for R if $i > \bar{x}$, with turnout probability $\tau(i) = \alpha \cdot 2\Delta|\bar{x} - i|$. The vote margin is:

$$\begin{aligned} V_{L,s} - V_{R,s} &= \int_{-\infty}^{\bar{x}} \alpha \cdot 2\Delta(\bar{x} - i)\, f_s(i)\, di - \int_{\bar{x}}^{\infty} \alpha \cdot 2\Delta(i - \bar{x})\, f_s(i)\, di \\ &= 2\alpha\Delta \left[\int_{-\infty}^{\bar{x}} (\bar{x} - i)\, f_s(i)\, di - \int_{\bar{x}}^{\infty} (i - \bar{x})\, f_s(i)\, di \right] \\ &= 2\alpha\Delta \int_{-\infty}^{\infty} (\bar{x} - i)\, f_s(i)\, di \\ &= 2\alpha\Delta(\bar{x} - \mu_s). \end{aligned}$$

The third equality combines both integrals into a single integral of $(\bar{x} - i) f_s(i)$ over the entire real line. The final equality uses $\int i\, f_s(i)\, di = \mu_s$. Note that the margin depends on the state mean μ_s but not on the variance σ^2—a consequence of the linear turnout function. □

2 Benchmark Marginal Returns and Proof of Proposition 1

I derive the marginal electoral returns $\Gamma^{\text{NT}}_{\text{PV}}$ and $\Gamma^{\text{NT}}_{\text{EC}}$ for the benchmark model without turnout effects and verify that they depend only on model parameters $(\mu, \sigma, \sigma_\omega)$, not on the equilibrium platform x^*.

Without endogenous turnout, all voters turn out with certainty. The vote share for L in state s is $F_s(\bar{x}) = \Phi((\bar{x} - \mu_s)/\sigma)$, giving vote margin $M_s^{\text{NT}} = 2\Phi((\bar{x} - \mu_s)/\sigma) - 1$. At a symmetric profile ($\bar{x} = 0$), the margin sensitivity is

$$D_s^{\text{NT}} \equiv \left. \frac{\partial M_s^{\text{NT}}}{\partial x_L} \right|_{\bar{x}=0} = \frac{\phi(\mu_s/\sigma)}{\sigma},$$

using $\partial \bar{x}/\partial x_L = 1/2$ and $\phi(-z) = \phi(z)$. Crucially, D_s^{NT} depends on μ_s and σ but not on x^*. Let $\rho \equiv \mu/\sigma$. Then $D_\ell^{\text{NT}} = D_r^{\text{NT}} = \phi(\rho)/\sigma$ and $D_m^{\text{NT}} = \phi(0)/\sigma = 1/(\sigma\sqrt{2\pi})$.

Popular Vote. At a symmetric profile, $\sum_s M_s^{\text{NT}} = 0$ (by the symmetry $\mu_\ell = -\mu_r$ and $\mu_m = 0$), and the aggregate shock has variance $3\sigma_\omega^2$. Therefore

$$\Gamma_{\text{PV}}^{\text{NT}} = \frac{\phi(0)}{\sqrt{3}\,\sigma_\omega} \sum_s D_s^{\text{NT}} = \frac{1}{\sqrt{3}\,\sigma_\omega \sqrt{2\pi}} \cdot \frac{1 + 2e^{-\rho^2/2}}{\sigma\sqrt{2\pi}} = \frac{1 + 2e^{-\rho^2/2}}{2\pi\sqrt{3}\,\sigma_\omega \sigma}.$$

This is independent of x^* and exact for all ρ and σ_ω.

Electoral College. The EC winning probability is $P_{\text{EC}} = \pi_\ell \pi_m + \pi_\ell \pi_r + \pi_m \pi_r - 2\pi_\ell \pi_m \pi_r$, where $\pi_s = \Phi(M_s^{\text{NT}}/\sigma_\omega)$. At a symmetric profile, $\pi_m = 1/2$, $\pi_\ell = \Phi(a)$, and $\pi_r = 1 - \Phi(a)$, with

$$a \equiv \frac{M_\ell^{\text{NT}}}{\sigma_\omega} = \frac{2\Phi(\rho) - 1}{\sigma_\omega}.$$

The partial derivatives of P_{EC} at a symmetric profile are

$$\frac{\partial P_{\text{EC}}}{\partial \pi_\ell} = \frac{\partial P_{\text{EC}}}{\partial \pi_r} = \frac{1}{2}, \qquad \frac{\partial P_{\text{EC}}}{\partial \pi_m} = \Phi(a)^2 + (1 - \Phi(a))^2.$$

Since $\partial \pi_s / \partial x_L = (\phi(M_s^{\text{NT}}/\sigma_\omega)/\sigma_\omega) \cdot D_s^{\text{NT}}$ and $D_\ell^{\text{NT}} = D_r^{\text{NT}}$, the chain rule gives

$$\Gamma_{\text{EC}}^{\text{NT}} = \frac{\phi(a)\,\phi(\rho)}{\sigma_\omega \sigma} + \frac{\Phi(a)^2 + (1 - \Phi(a))^2}{2\pi \sigma_\omega \sigma}.$$

Again, this depends only on ρ and σ_ω, not on x^*. The first term captures safe states' direct contribution; the second captures the swing state's pivotality, weighted by $\Phi(a)^2 + (1 - \Phi(a))^2 \in [1/2, 1]$, which reflects the probability that the safe states split (making the swing state decisive).

Proof of Proposition 1. Let $R(\rho) \equiv \Gamma_{\text{PV}}^{\text{NT}} / \Gamma_{\text{EC}}^{\text{NT}}$, so PV produces less divergence iff $R > 1$.

Part (a). At $\rho = 0$, all states are identical: $a = 0$, $\Phi(a) = 1/2$, $\phi(a) = \phi(0)$, and $R(0) = 2\sqrt{3}/3 > 1$.

Part (b). As $\rho \to \infty$: $\phi(\rho) \to 0$ (safe-state sensitivity vanishes) and $a \to 1/\sigma_\omega$. Define

$p_\infty \equiv \Phi(1/\sigma_\omega)$. Then

$$R(\rho) \;\to\; \frac{1}{\sqrt{3}\,[p_\infty^2 + (1-p_\infty)^2]}.$$

This limit is less than 1 iff $p_\infty^2 + (1-p_\infty)^2 > 1/\sqrt{3}$, i.e., when $\sigma_\omega < \bar{\sigma}_\omega$, where $\bar{\sigma}_\omega$ solves $\Phi(1/\bar{\sigma}_\omega)^2 + (1-\Phi(1/\bar{\sigma}_\omega))^2 = 1/\sqrt{3}$, giving $\bar{\sigma}_\omega \approx 1.94$. Since R is continuous with $R(0) > 1$ and $R(\infty) < 1$, by the intermediate value theorem a threshold $\bar{\rho}$ exists.

Part (c). When $\sigma_\omega \geq \bar{\sigma}_\omega$, $R(\infty) \geq 1$ and $R(\rho) > 1$ for all finite ρ. □

Extended interpretation of Proposition 1. The following discussion, referenced in the main text, provides a more detailed interpretation of each part.

Under the popular vote, a candidate's margin of victory reflects every vote cast across the country, so a platform shift moves the national margin through all states simultaneously. Under the Electoral College, what matters is winning states: a platform shift helps only insofar as it flips competitive states.

When states are similar (part a), each state produces a noisy win-or-lose outcome. Aggregating these binary outcomes is less responsive to platform changes than simply counting all votes, so candidates face a weaker incentive to moderate under EC than under PV.

When states are highly polarized and state-level shocks are small (part b), the intuition reverses. Safe states become electorally inert: few voters lie near the partisan cutpoint, so platform changes barely move the vote margin there. Yet under PV, random shocks from these inert states still enter the national vote margin, weakening the connection between candidates' positions and who wins. The Electoral College, by effectively discarding safe-state margins, concentrates the election on the swing state, where moderation moves the vote margin without being diluted by noise from inert states.

Part (c) identifies when this reversal cannot occur. When state shocks are large ($\sigma_\omega \geq 1.94$), even nominally safe states become coin flips, and the distinction between safe and swing states effectively disappears. The threshold $\bar{\sigma}_\omega \approx 1.94$ is large relative to the normalized state-level vote-margin scale, corresponding to a situation where election outcomes are dominated by random shocks rather than fundamentals.

3 Proof of Proposition 2

From equations (6) and (7) in the main text, $\Gamma_{\text{PV}} = \sqrt{3}\,\gamma_0\,x^*$ and $\Gamma_{\text{EC}} = \gamma_0\,f(\lambda)\,x^*$, so $\Gamma_{\text{PV}}/\Gamma_{\text{EC}} = \sqrt{3}/f(\lambda)$.

I need to verify that $f(\lambda) < \sqrt{3}$ for all $\lambda \geq 0$. Since $f(0) = e^0 + \Phi(0)^2 + (1-\Phi(0))^2 = 1 + 1/4 + 1/4 = 3/2$ and f is decreasing (see Section 8), $f(\lambda) \leq f(0) = 3/2 < \sqrt{3} \approx 1.732$ for all $\lambda \geq 0$. Therefore the ratio exceeds one.

For the equilibrium comparison, note that the PV first-order condition (equation (3) in the main text) is the quadratic $4\beta\gamma_{\text{PV}}(x^*)^2 + x^* - \beta = 0$, which is strictly increasing in x^* for $x^* > 0$. Evaluating the left side at the EC equilibrium x^*_{EC}:

$$4\beta\gamma_{\text{PV}}(x^*_{\text{EC}})^2 + x^*_{\text{EC}} - \beta = 4\beta\gamma_0(\sqrt{3} - f(\lambda))(x^*_{\text{EC}})^2 > 0,$$

where I used the EC first-order condition $4\beta\gamma_{\text{EC}}(x^*_{\text{EC}})^2 + x^*_{\text{EC}} - \beta = 0$. Since the PV left side is positive at x^*_{EC} and zero at x^*_{PV}, it follows that $x^*_{\text{PV}} < x^*_{\text{EC}}$.

The ratio $\sqrt{3}/f(\lambda)$ is strictly increasing in λ (and hence in μ) because f is strictly decreasing (Section 8). The bounds follow from $f(0) = 3/2$ (giving $2\sqrt{3}/3$) and $\lim_{\lambda\to\infty} f(\lambda) = 1$ (giving $\sqrt{3}$). □

4 Proof of Corollary 1

With $n = 2K + 1$ states, one swing state with $\mu_m = 0$, and K safe states on each side, the total margin sensitivity at a symmetric equilibrium is $\sum_s D_s = 2n\alpha x^*$ (since safe-state sensitivities pair as $D_s + D_{s'} = 4\alpha x^*$ for each symmetric pair, plus $D_m = 2\alpha x^*$). The aggregate shock has variance $n\sigma_\omega^2$, so the PV marginal return is

$$\Gamma_{\text{PV}}^{(n)} = \frac{\phi(0)}{\sqrt{n}\,\sigma_\omega} \cdot 2n\alpha x^* = \frac{2\sqrt{n}\,\alpha x^*}{\sigma_\omega\sqrt{2\pi}} = \sqrt{n}\,\gamma_0\,x^*.$$

In the safe-state limit, $\Gamma_{\text{EC}} = \gamma_0\,x^*$ (independent of n), so $\Gamma_{\text{PV}}^{(n)} = \sqrt{n}\,\Gamma_{\text{EC}}$.

The PV equilibrium solves $4\beta\sqrt{n}\,\gamma_0\,(x^*)^2 + x^* - \beta = 0$, giving

$$x_{\text{PV}}^*(n) = \frac{-1 + \sqrt{1 + 16\beta^2\sqrt{n}\,\gamma_0}}{8\beta\sqrt{n}\,\gamma_0}.$$

As $n \to \infty$: the numerator grows as $\sqrt[4]{n}$ while the denominator grows as $\sqrt{n}$, so $x_{\text{PV}}^*(n) \to 0$. The EC equilibrium $x_{\text{EC}}^* = (-1 + \sqrt{1 + 16\beta^2\gamma_0})/(8\beta\gamma_0)$ is independent of n. □

5 Second-Order Conditions

I verify that the symmetric equilibrium characterized in Proposition 2 satisfies the second-order condition for both electoral systems.

At the symmetric equilibrium, the second derivative of $\mathbb{E}[U_L]$ with respect to x_L is

$$\frac{\partial^2 \mathbb{E}[U_L]}{\partial x_L^2} = \frac{\partial^2 P}{\partial x_L^2} \cdot 4\beta x^* - 4\Gamma(\beta - x^*) - 1. \tag{1}$$

I show that $\frac{\partial^2 P}{\partial x_L^2}\big|_{\text{sym}} = -\gamma_j$ for each system, where $\gamma_{\text{EC}} = 2\alpha/(\sigma_\omega\sqrt{2\pi})$ and $\gamma_{\text{PV}} = \sqrt{3}\,\gamma_{\text{EC}}$.

Electoral College. In the safe-state limit, $P_{\text{EC}} \approx \Phi(M_m/\sigma_\omega)$. At the symmetric equilibrium, $M_m = \alpha(x_R^2 - x_L^2)$, so

$$\begin{aligned}
\frac{\partial P}{\partial x_L} &= \frac{\phi(M_m/\sigma_\omega)}{\sigma_\omega} \cdot (-2\alpha x_L), \\
\frac{\partial^2 P}{\partial x_L^2} &= \frac{\phi'(M_m/\sigma_\omega)}{\sigma_\omega^2} \cdot (-2\alpha x_L)^2 + \frac{\phi(M_m/\sigma_\omega)}{\sigma_\omega} \cdot (-2\alpha).
\end{aligned}$$

At $x_L = -x^*$ and $M_m = 0$: $\phi'(0) = 0$ and $\phi(0) = 1/\sqrt{2\pi}$, so

$$\left.\frac{\partial^2 P}{\partial x_L^2}\right|_{\text{sym}} = -\frac{2\alpha}{\sigma_\omega\sqrt{2\pi}} = -\gamma_{\text{EC}}.$$

National Popular Vote. $P_{\text{PV}} = \Phi(S/(\sqrt{n}\,\sigma_\omega))$ where $S = \sum_s M_s = n\alpha(x_R^2 - x_L^2)$. Then

$$\frac{\partial P}{\partial x_L} = \frac{\phi(S/(\sqrt{n}\,\sigma_\omega))}{\sqrt{n}\,\sigma_\omega} \cdot (-2n\alpha x_L),$$
$$\frac{\partial^2 P}{\partial x_L^2} = \frac{\phi'(S/(\sqrt{n}\,\sigma_\omega))}{n\sigma_\omega^2} \cdot (-2n\alpha x_L)^2 + \frac{\phi(S/(\sqrt{n}\,\sigma_\omega))}{\sqrt{n}\,\sigma_\omega} \cdot (-2n\alpha).$$

At the symmetric equilibrium ($x_L = -x^*$, $S = 0$): again $\phi'(0) = 0$, so

$$\left.\frac{\partial^2 P}{\partial x_L^2}\right|_{\text{sym}} = -\frac{2n\alpha}{\sqrt{n}\,\sigma_\omega\sqrt{2\pi}} = -\frac{2\sqrt{n}\,\alpha}{\sigma_\omega\sqrt{2\pi}} = -\gamma_{\text{PV}}.$$

Verification. Substituting $\Gamma_j = \gamma_j x^*$ and $\frac{\partial^2 P}{\partial x_L^2} = -\gamma_j$ into (1):

$$\frac{\partial^2 \mathbb{E}[U_L]}{\partial x_L^2} = -4\gamma_j\beta x^* - 4\gamma_j x^*(\beta - x^*) - 1 = -4\gamma_j x^*(2\beta - x^*) - 1 < 0,$$

since $\gamma_j > 0$ and $x^* \in (0, \beta)$ implies $2\beta - x^* > 0$. The second-order condition holds for both electoral systems. □

6 Equilibrium Uniqueness

I show that the symmetric equilibrium identified in Proposition 2 is the *unique* Nash equilibrium of the platform-choice game.

Step 1: Win probability structure. Under both electoral systems, the win probability in the safe-state limit takes the form

$$P(x_L, x_R) = \Phi\big(\tilde{c}(x_R^2 - x_L^2)\big)$$

for a constant $\tilde{c} > 0$. To see this, note that the vote margin in the swing state is $M_m = 2\alpha\Delta(\bar{x} - 0) = 2\alpha(x_R - x_L) \cdot \frac{x_L + x_R}{2} = \alpha(x_R^2 - x_L^2)$. Under EC with safe states, $P_{\text{EC}} = \Phi(M_m/\sigma_\omega) = \Phi(\alpha(x_R^2 - x_L^2)/\sigma_\omega)$, so $\tilde{c}_{\text{EC}} = \alpha/\sigma_\omega$. Under PV, $\sum_s M_s = n\alpha(x_R^2 - x_L^2)$, so $P_{\text{PV}} = \Phi(\sqrt{n}\,\alpha(x_R^2 - x_L^2)/\sigma_\omega)$, giving $\tilde{c}_{\text{PV}} = \sqrt{n}\,\alpha/\sigma_\omega$.

Step 2: Log-concavity of the win probability. Define $g(x_L) \equiv \tilde{c}(x_R^2 - x_L^2)$. This function is concave in x_L (since $g''(x_L) = -2\tilde{c} < 0$). The standard normal CDF Φ is log-concave and increasing on $\mathbb{R}$. The composition of an increasing log-concave function with a concave function is log-concave (Prékopa, 1973). Therefore $P(x_L, x_R) = \Phi(g(x_L))$ is log-concave in x_L for each fixed x_R.

Step 3: Global concavity of expected utility. Candidate L's expected utility is

$$\mathbb{E}[U_L] = P \cdot [-(x_L + \beta)^2] + (1 - P) \cdot [-(x_R + \beta)^2].$$

Rearranging: $\mathbb{E}[U_L] = -(x_R + \beta)^2 + P \cdot [(x_R + \beta)^2 - (x_L + \beta)^2]$. The term $(x_R + \beta)^2 - (x_L + \beta)^2$ is positive for $x_L < x_R$ (the relevant range) and concave in x_L. Since P is log-concave and positive, the product is concave in x_L on the relevant domain. (More directly, the SOC verification in Section 5 shows that $\partial^2 \mathbb{E}[U_L]/\partial x_L^2 < 0$ at any interior critical point, ensuring that any critical point is a global maximum.)

Step 4: Unique best response. Since $\mathbb{E}[U_L]$ is strictly concave in x_L on the interior of the relevant domain, each candidate has a unique best response to any strategy of the opponent.

Step 5: Symmetry implies unique equilibrium. The game is symmetric: if (x_L^*, x_R^*) is a Nash equilibrium, then $(-x_R^*, -x_L^*)$ is also a Nash equilibrium (by the mirror symmetry of preferences and state structure). Since the best response is unique and continuous, the unique fixed point of the best-response mapping is the symmetric profile $x_L = -x^*$, $x_R = x^*$, where x^* is the unique positive root of equation (8) in the main text (Calvert, 1985).

For EC with arbitrary μ (outside the safe-state limit), the nonlinear structure of P_{EC}—a polynomial of three normal CDFs—precludes a closed-form global concavity argument. A similar difficulty applies to the benchmark model without endogenous turnout. I verify uniqueness numerically for these cases in Section 11. □

7 Asymmetric States

The main text assumes symmetric state leanings ($\mu_\ell = -\mu$, $\mu_r = \mu$). I now relax this assumption and allow asymmetric partisan geography.

Consider n states with means $\mu_1, \ldots, \mu_n$, one of which is a swing state with $\mu_m = 0$. Define the *partisan imbalance* $S_\mu \equiv \sum_{s=1}^{n} \mu_s$. When $S_\mu = 0$, the state means are balanced (the symmetric case); when $S_\mu > 0$, there is a net rightward lean.

From Lemma 1, the margin sensitivity in state s at the symmetric equilibrium is $D_s = 2\alpha(\mu_s + x^*)$. Under EC (swing state decisive):

$$\Gamma_{\text{EC}}^L = \frac{\phi(0)}{\sigma_\omega} \cdot D_m = \frac{\phi(0)}{\sigma_\omega} \cdot 2\alpha|x_L|,$$

where I used $D_m = 2\alpha(0 + x^*) = 2\alpha|x_L|$ at the symmetric equilibrium. Under PV, the total sensitivity is $\sum_s D_s = 2\alpha(\sum_s \mu_s + nx^*) = 2\alpha(S_\mu + n|x_L|)$, and the aggregate shock has standard deviation $\sqrt{n}\,\sigma_\omega$, so

$$\Gamma_{\text{PV}}^L = \frac{\phi(0)}{\sqrt{n}\,\sigma_\omega} \cdot 2\alpha(S_\mu + n|x_L|).$$

The ratio of marginal electoral returns for candidate L is therefore

$$\frac{\Gamma^L_{\text{PV}}}{\Gamma^L_{\text{EC}}} = \frac{S_\mu + n|x_L|}{\sqrt{n}\,|x_L|} = \sqrt{n} + \frac{S_\mu}{\sqrt{n}\,|x_L|}. \tag{2}$$

By symmetry of the argument (replacing S_μ with $-S_\mu$ for candidate R, since a rightward lean reduces R's opposition-demobilization benefit), the ratio for candidate R is

$$\frac{\Gamma^R_{\text{PV}}}{\Gamma^R_{\text{EC}}} = \sqrt{n} - \frac{S_\mu}{\sqrt{n}\,x_R}. \tag{3}$$

Equations (2)–(3) reveal three features of the asymmetric case:

1. *Symmetric benchmark.* When $S_\mu = 0$, both ratios equal $\sqrt{n}$, recovering Proposition 2 and Corollary 1.

2. *Asymmetric amplification.* When $S_\mu > 0$ (net rightward lean), the PV moderation effect is amplified for L (who faces more opposing safe states) and attenuated for R (who faces fewer). The correction terms are equal in magnitude but opposite in sign, reflecting that partisan imbalance redistributes the opposition-demobilization pressure across candidates without eliminating it.

3. *Robustness.* Both ratios exceed one—so PV produces more moderation than EC for *both* candidates—as long as the partisan imbalance is not too extreme. Specifically, $\Gamma^R_{\text{PV}}/\Gamma^R_{\text{EC}} > 1$ requires

   $$S_\mu < (n - \sqrt{n})\,x_R.$$

   For $n = 51$ and $x_R \approx 0.5$, this allows $S_\mu < 21.9$—a very permissive condition even with substantial partisan asymmetry.

The key insight is that partisan asymmetry tilts the opposition-demobilization channel toward one candidate but does not shut it down for either. Even the candidate with fewer opposing safe states faces strictly greater moderation pressure under PV than under EC, provided the overall partisan imbalance is moderate.

8 Exact EC Marginal Return and Monotonicity of f

This section provides the derivation of the EC marginal electoral return Γ_{EC} used in Proposition 2 and verifies the monotonicity of $f(\lambda)$.

Derivation. At the symmetric equilibrium, $\pi_\ell = \Phi(\lambda)$, $\pi_m = \frac{1}{2}$, and $\pi_r = 1 - \Phi(\lambda)$, where $\lambda \equiv 4\alpha x^* \mu/\sigma_\omega \geq 0$. The partial derivatives of P_{EC} with respect to the state-winning probabilities are

$$\frac{\partial P_{\text{EC}}}{\partial \pi_\ell} = \frac{\partial P_{\text{EC}}}{\partial \pi_r} = \frac{1}{2}, \qquad \frac{\partial P_{\text{EC}}}{\partial \pi_m} = 1 - 2\Phi(\lambda)\big(1 - \Phi(\lambda)\big).$$

Since $\partial \pi_s/\partial x_L\big|_{\text{sym}} = (\phi(c_s)/\sigma_\omega) \cdot D_s$ with $\phi(c_\ell) = \phi(c_r) = \phi(\lambda)$ and $\phi(c_m) = \phi(0)$, the chain rule gives

$$\Gamma_{\text{EC}} = \frac{\phi(\lambda)}{2\sigma_\omega}(D_\ell + D_r) + \frac{\phi(0)}{\sigma_\omega}\big[1 - 2\Phi(\lambda)(1 - \Phi(\lambda))\big] D_m. \tag{4}$$

Substituting $D_\ell + D_r = 4\alpha x^*$, $D_m = 2\alpha x^*$, and $\phi(\lambda) = \phi(0)\, e^{-\lambda^2/2}$:

$$\Gamma_{\text{EC}} = \frac{2\alpha x^*\, \phi(0)}{\sigma_\omega} \cdot f(\lambda) = \gamma_0\, f(\lambda)\, x^*,$$

where $f(\lambda) \equiv e^{-\lambda^2/2} + 1 - 2\Phi(\lambda)\big(1 - \Phi(\lambda)\big) = e^{-\lambda^2/2} + \Phi(\lambda)^2 + (1 - \Phi(\lambda))^2$ as stated in the main text.

Monotonicity of f. I show that f is strictly decreasing from $f(0) = 3/2$ to $\lim_{\lambda\to\infty} f(\lambda) = 1$. Differentiating:

$$f'(\lambda) = e^{-\lambda^2/2}\, h(\lambda), \qquad h(\lambda) \equiv -\lambda + \frac{2}{\sqrt{2\pi}}\big(2\Phi(\lambda) - 1\big).$$

Since $h(0) = 0$ and $h'(\lambda) = -1 + (2/\pi)\, e^{-\lambda^2/2} \leq -1 + 2/\pi < 0$ for all $\lambda \geq 0$, the function h is strictly negative for $\lambda > 0$. Therefore $f'(\lambda) < 0$, confirming that f is strictly decreasing. The ratio $\Gamma_{\text{PV}}/\Gamma_{\text{EC}} = \sqrt{3}/f(\lambda)$ satisfies

$$\frac{2\sqrt{3}}{3} \;\leq\; \frac{\Gamma_{\text{PV}}}{\Gamma_{\text{EC}}} \;\leq\; \sqrt{3}, \tag{5}$$

with the lower bound $2\sqrt{3}/3 \approx 1.155$ attained when $\mu = 0$ (all states identical) and the upper bound approached as $\mu \to \infty$ (safe-state limit). □

9 Additional Comparative Statics

From the closed-form equilibrium (equation (8) in the main text), I derive comparative statics with respect to the model parameters.

Define $k \equiv 8\alpha\beta/(\sigma_\omega\sqrt{2\pi})$. The equilibria are

$$x^*_{\text{EC}} = \frac{-1 + \sqrt{1 + 4k\beta}}{2k}, \tag{6}$$

$$x^*_{\text{PV}}(n) = \frac{-1 + \sqrt{1 + 4\sqrt{n}\,k\beta}}{2\sqrt{n}\,k}. \tag{7}$$

Policy motivation (β). Both x^*_{EC} and x^*_{PV} are strictly increasing in β. As $\beta \to 0$, both converge to $x^* = 0$ (Downsian convergence). As $\beta \to \infty$:

$$x^*_j \approx \sqrt{\frac{\beta}{4\beta\gamma_j}} = \frac{1}{\sqrt{4\gamma_j}} \cdot \beta^0 \to \frac{1}{2\sqrt{\gamma_j}},$$

so divergence grows without bound but $x^*/\beta \to 0$—candidates never reach their ideal points.

Turnout sensitivity (α). Since k is proportional to α, higher turnout sensitivity increases k, which reduces x^* under both systems. The reduction is proportionally larger under PV because $\gamma_{\text{PV}} = \sqrt{n}\,\gamma_{\text{EC}}$. Intuitively, higher turnout sensitivity amplifies the electoral return to moderation, and this amplification is itself amplified under PV by the $\sqrt{n}$ factor.

Electoral uncertainty (σ_ω). Since k is inversely proportional to σ_ω, greater electoral uncertainty decreases k, which increases x^* under both systems. In the limit $\sigma_\omega \to \infty$ (maximal uncertainty): $k \to 0$, so $x_j^* \to \beta$ (candidates announce their ideal points, since the election outcome is essentially a coin flip regardless of platforms). In the limit $\sigma_\omega \to 0$ (no uncertainty): $k \to \infty$, so $x_j^* \to 0$ (candidates converge to the median, since even tiny moderation produces a large swing in win probability).

Divergence ratio. The ratio $x_{\text{PV}}^*/x_{\text{EC}}^*$ is bounded:

$$\frac{1}{\sqrt[4]{n}} < \frac{x_{\text{PV}}^*(n)}{x_{\text{EC}}^*} < 1$$

for all parameter values. The lower bound is approached as $\beta \to \infty$ (since $x_j^* \propto \gamma_j^{-1/2}$, the ratio approaches $(\gamma_{\text{EC}}/\gamma_{\text{PV}})^{1/2} = n^{-1/4}$). The upper bound is approached as $\beta \to 0$. For $n = 51$, this gives a range of $(0.37, 1)$, consistent with the numerical illustrations in Figure 1.

10 Correlated Shocks: General Case

The main text (Section 6.3) derives a closed-form expression for the marginal-return ratio under correlated shocks in the safe-state limit. In that limit the EC election is decided by the swing state alone, so the correlation structure affects only the PV side and the ratio takes the simple form $\sqrt{n/(1+(n-1)\rho)}$.

Outside the safe-state limit, the correlation also changes the EC winning probability. With independent shocks, $P_{\text{EC}} = \pi_\ell\pi_m + \pi_\ell\pi_r + \pi_m\pi_r - 2\pi_\ell\pi_m\pi_r$ because state outcomes are independent. With correlated shocks ($\omega_s = \eta + \varepsilon_s$, $\text{Corr}(\omega_s, \omega_{s'}) = \rho$), the joint probability of winning two or more states involves bivariate and trivariate normal CDFs:

$$P_{\text{EC}} = \Phi_2(a_\ell, a_m; \rho) + \Phi_2(a_\ell, a_r; \rho) + \Phi_2(a_m, a_r; \rho) - 2\Phi_3(a_\ell, a_m, a_r; \Sigma_\rho),$$

where $a_s \equiv M_s/\sigma_\omega$ and Σ_ρ is the equicorrelation matrix with off-diagonal entries ρ. Individual state win probabilities $\pi_s = \Phi(a_s)$ are unchanged (since $\text{Var}(\omega_s) = \sigma_\omega^2$ regardless of ρ), but the joint distribution changes. A closed-form comparison of marginal returns is therefore unavailable in the general case.

Method. I verify numerically that PV produces strictly less platform divergence for all $\rho < 1$. For each parameter combination, I solve for the symmetric equilibrium x^* under both systems. The PV equilibrium admits a closed form even with correlated shocks: since the aggregate shock has variance $n\sigma_\omega^2[1 + (n-1)\rho]$, the marginal return is $\Gamma_{\text{PV}} = \sqrt{3}\,\gamma_0\, x^*/\sqrt{1+2\rho}$ and the FOC is a quadratic in x^*. For EC, I compute Γ_{EC} using the analytical derivatives of the multivariate normal CDF:

$$\frac{\partial \Phi_2(a, b; \rho)}{\partial a} = \phi(a)\,\Phi\left(\frac{b - \rho a}{\sqrt{1-\rho^2}}\right),$$

$$\frac{\partial \Phi_3(a_1, a_2, a_3; \Sigma_\rho)}{\partial a_1} = \phi(a_1)\,\Phi_2\left(\frac{a_2 - \rho a_1}{\sqrt{1-\rho^2}},\ \frac{a_3 - \rho a_1}{\sqrt{1-\rho^2}};\ \frac{\rho}{1+\rho}\right),$$

and solve the FOC $\Gamma_{\text{EC}}(x^*) \cdot 4\beta x^* + x^* - \beta = 0$ via root-finding.

Results. I evaluate 1,080 parameter combinations: $\mu \in \{0.1, 0.5, 1, 2, 3, 5\}$, $\sigma_\omega \in \{0.5, 1, 2, 3\}$, $\beta \in \{0.5, 1, 2\}$, $\alpha \in \{0.5, 1, 2\}$, $\rho \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$. In every case, $x^*_{\text{PV}} < x^*_{\text{EC}}$: the popular vote produces strictly less platform divergence. As expected, the ratio $x^*_{\text{PV}}/x^*_{\text{EC}}$ increases with ρ (i.e., the gap narrows as shocks become more correlated), with a median ratio ranging from 0.90 at $\rho = 0.1$ to 0.99 at $\rho = 0.9$.

11 Numerical Verification of Equilibrium Uniqueness

Section 6 provides analytical uniqueness proofs for the safe-state limit under both systems, and for PV with endogenous turnout for all μ. For the remaining cases—EC with endogenous turnout at arbitrary μ, and both systems under the benchmark without turnout—the win probability has a more complex structure that precludes a closed-form concavity argument. I verify uniqueness numerically for all of these cases.

Method. For each parameter combination, I compute candidate L's best response $BR_L(x_R) = \arg\max_{x_L} \mathbb{E}[U_L(x_L, x_R)]$ over a fine grid of x_R values using numerical optimization. A Nash equilibrium corresponds to a fixed point of the best-response mapping. By the game's mirror symmetry, symmetric equilibria satisfy $BR_L(x_R) = -x_R$. I check for asymmetric equilibria by counting crossings of $BR_L(x_R)$ with the mirror line $x_L = -x_R$.

No-turnout benchmark. I evaluate 216 parameter combinations: $\mu \in \{0.1, 0.5, 1, 2, 3, 5\}$, $\sigma_\omega \in \{0.5, 1, 2, 3\}$, $\beta \in \{0.5, 1, 2\}$, $\sigma \in \{0.5, 1, 2\}$. In every case the best-response curve crosses the mirror line exactly once under both systems, confirming that the symmetric equilibrium is the unique Nash equilibrium. Figure 1 displays representative cases.

Endogenous turnout model. I evaluate 216 parameter combinations: $\mu \in \{0.1, 0.5, 1, 2, 3, 5\}$, $\sigma_\omega \in \{0.5, 1, 2, 3\}$, $\beta \in \{0.5, 1, 2\}$, $\alpha \in \{0.5, 1, 2\}$, with $BR_L(x_R)$ computed at 200 points per combination. Again, in every case the best-response curve crosses the mirror line exactly once under both EC and PV, confirming uniqueness. Figure 2 displays representative cases. R replication files for all numerical results are available as supplementary materials.

Reference.

Calvert, R. L. (1985). "Robustness of the Multidimensional Voting Model: Candidate Motivations, Uncertainty, and Convergence." *American Journal of Political Science*, 29(1), 69–95.

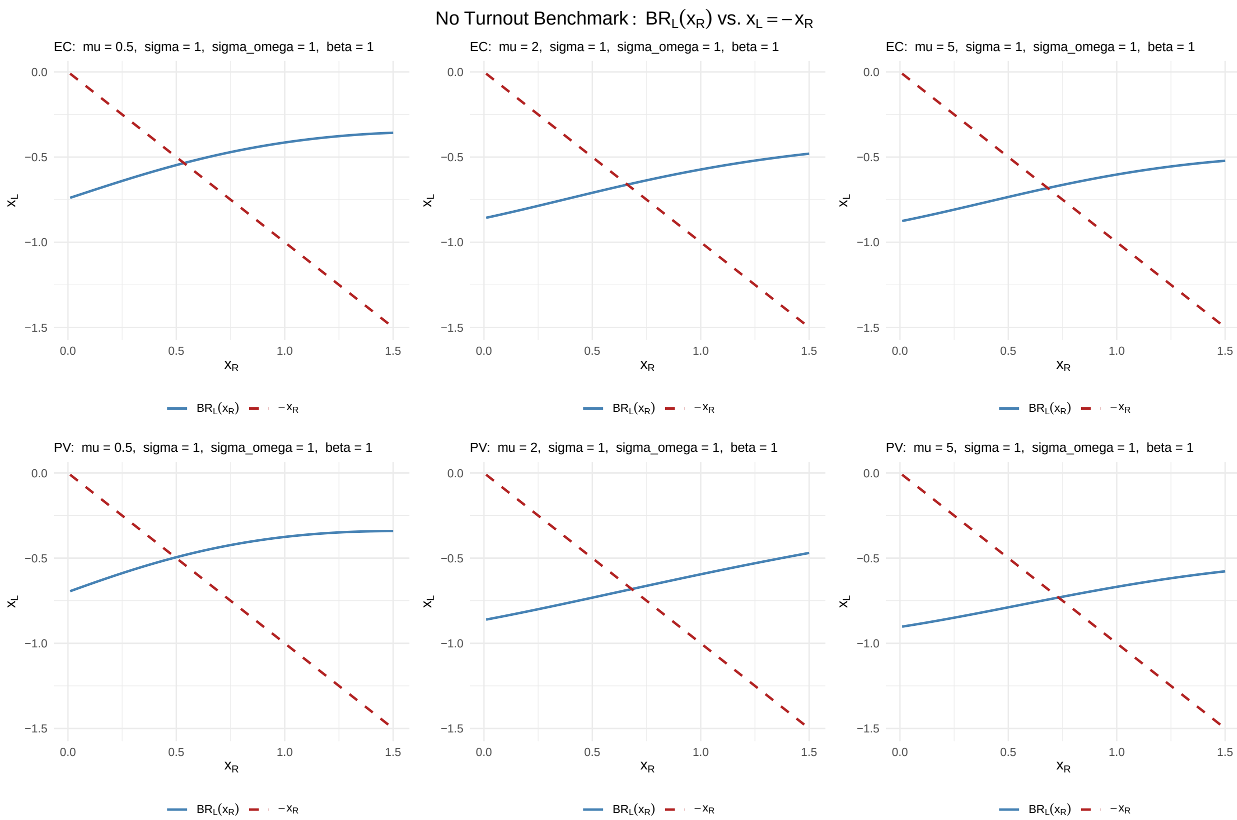

Figure 1: No-turnout benchmark. Best response $BR_L(x_R)$ (solid) and the mirror line $x_L = -x_R$ (dashed) under EC (top) and PV (bottom). In all cases there is exactly one crossing, confirming uniqueness. Parameters: $\sigma = 1$, $\sigma_\omega = 1$, $\beta = 1$.

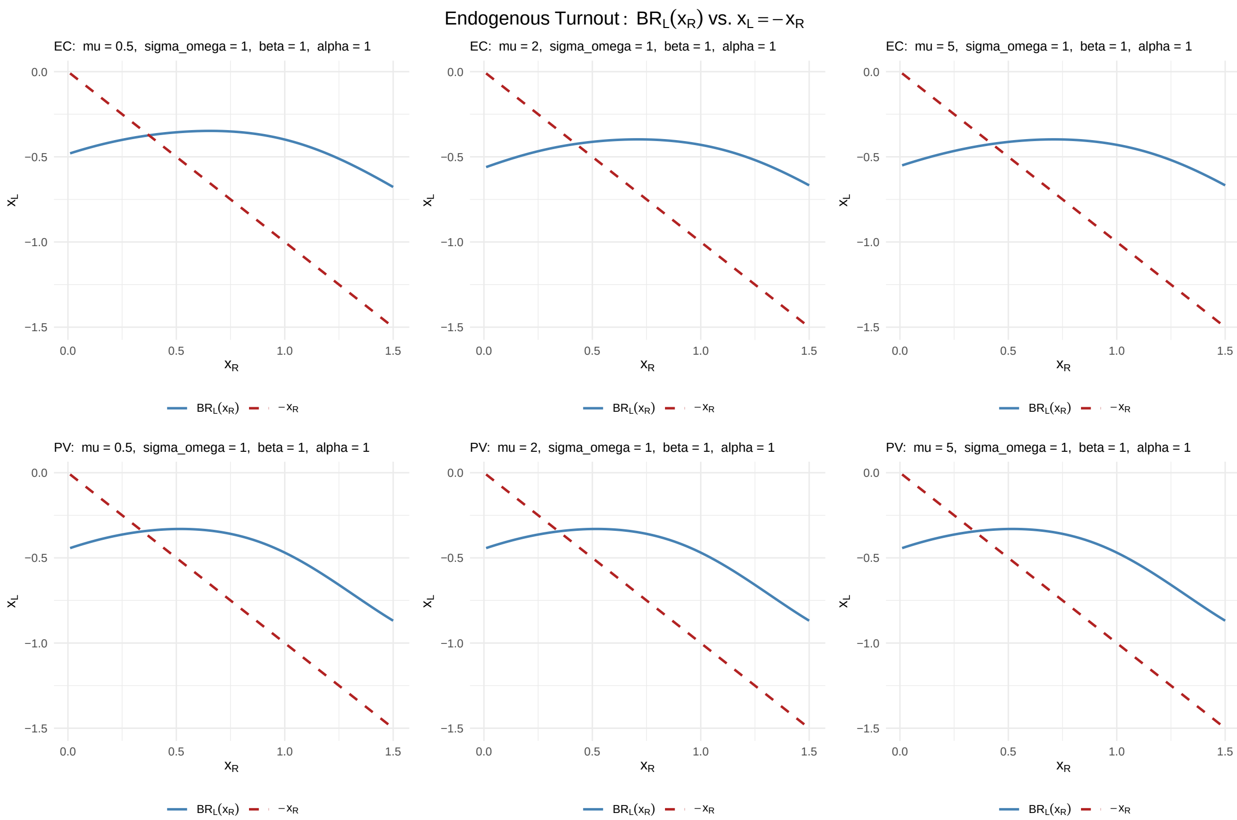

Figure 2: Endogenous turnout model. Best response $BR_L(x_R)$ (solid) and the mirror line $x_L = -x_R$ (dashed) under EC (top) and PV (bottom), for low ($\mu = 0.5$), moderate ($\mu = 2$), and high ($\mu = 5$) state polarization. Each crossing corresponds to a Nash equilibrium. In all cases there is exactly one crossing. Parameters: $\sigma_\omega = 1$, $\beta = 1$, $\alpha = 1$.

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